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Velocity measurements close to a rough plate oscillating in its own plane

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Measurements are described of the fluid velocities close to rough beds oscillating in their own plane. The roughness with which most of the results were obtained consisted of smooth spheres closely packed in hexagonal formation. Some results are also given for beds of gravel. The beds were oscillated with simple harmonic motion in still air and the measurements were made with a hot-wire anemometer.

The measurements very close to the beds of smooth spheres show two maxima in the velocity profile during each half-cycle. One maximum corresponds to a component of velocity which varies nearly sinusoidally with time. The second forms quite a sharp peak and occurs close to $\omega t = 90°$, $270°$, where ω is the angular frequency of oscillation and t is time measured from the instant of maximum velocity of the plate. The phase at which this peak occurs shows little variation with distance from the bed. For values of $\beta D > 3{\cdot}0$, where $\beta = (\omega/2\nu)^{\frac{1}{2}}$, ν is the kinematic viscosity and D is the sphere diameter, the maximum velocity during each half-cycle is found at this peak over at least a certain range of distances from the bed. The variation with height of the nearly sinusoidal component of velocity is quite close to that given by Stokes' (1851) solution for a flat plate. The peak at $\omega t = 90°$, $270°$, however, rises from zero at the bed to a maximum at a distance of about one-eighth of a sphere diameter above the crests and then falls off again.

The measurements with beds of gravel show a variation in velocity similar to that observed by Kalkanis (1957, 1964) and Sleath (1970). Because of the irregularity of the surface it is difficult to draw definite conclusions about the flow in the immediate vicinity of the bed.

A number of tests were carried out, with the beds of spheres, using a wire slanted at 45° to the bed in order to determine the velocity product uw.

1. Introduction

Although the theoretical solution for the velocity distribution above a flat plate oscillating in its own plane was given by Stokes in 1851, it is only relatively recently that oscillatory velocities above rough beds have been investigated.

Measurements above an oscillating tray covered with both two-dimensional and three-dimensional roughness elements have been made by Kalkanis (1957, 1964). Jonsson (1963) measured velocities in an oscillating water tunnel with two-dimensional bed roughness and Horikawa & Watanabe (1968) and Sleath (1970) have made measurements close to rough beds in wave tanks. It is well known that relative to axes fixed in the plate the velocity distribution above a plate oscillating in its own plane is the same as that produced by water waves above a stationary plate, provided that both the plate and the length of the waves may be assumed infinite.

These measurements show that the velocity distribution above a rough bed may be significantly different from that given by Stokes' solution. Unfortunately, all of the instruments used to measure velocity were bulky and consequently little can be deduced about the velocities in the immediate vicinity of the bed roughness. One of the objects of the work described in this paper was to make detailed measurements very close to three-dimensionally rough beds. These measurements are described in §3.

Attempts to calculate turbulent oscillatory flow over rough beds using assumptions about eddy viscosity have been made by Jonsson (1966) and Kajiura (1968), but the results of Horikawa & Watanabe (1968) suggest that the assumptions made are incorrect. However, there is very little experimental evidence one way or the other. Consequently an attempt has also been made to measure the product of the principal components of the velocity with a view to shedding some light on the distribution of Reynolds stress. These measurements are discussed in §4.

2. Test conditions and equipment

Most of the tests were made with beds consisting of a layer of smooth spheres packed close together in a hexagonal array on a flat plate of length 1·67 m and width 0·30 m. The spheres were aligned in rows parallel to the short edges of the plate. Five different beds were used, with sphere diameters of 3·10, 6·32, 12·34, 18·60 and 37·80 mm. The 6·32 mm spheres covered only the central portion of the plate over half the width and half the length. In all other cases the spheres covered the whole of the plate. The advantage of this very regular roughness is that it facilitates interpretation of the velocity records and ensures repeatable conditions from one test to the next. However, natural beds of sand and gravel are decidedly irregular and consequently a number of tests were made with two different grades of gravel glued over the whole of the flat plate. The larger grade passed through a 12·70 mm sieve and was retained on a 9·53 mm sieve. The smaller passed through a 9·53 mm sieve and was retained by a 4·76 mm sieve.

The plate was mounted on rollers and was caused to oscillate with simple harmonic motion, parallel to its longer edge, by a variable-speed motor with feedback control driving a scotch yoke. The distance between successive spheres in the direction of oscillation was 1·73 diameters and at right angles was one diameter. The plate was totally enclosed in a tank whose dimensions were $2{\cdot}14 \times 0{\cdot}33 \times 0{\cdot}91$ m and the clearance between the plate and the bottom of the tank was 0·25 m. At 0·4 m from each end of the tank vertical baffles extended from the top of the tank to within 20 or 30 mm of the bed. The purpose of these baffles was to prevent vortices shed from the ends of the bed from affecting the flow near the centre and to impede any gross oscillation of the air in the tank due to the motion of the plate and driving mechanism. With these baffles in place the amplitude of the velocity outside the viscous boundary layer was found not to exceed 2 % of the amplitude of oscillation of the bed.

All of the measurements were made in air as this facilitated the use of a hot-wire anemometer. The instrument used was a DISA 55A01 constant-temperature anemometer with 55A25 and 55A29 miniature probes. Output from the anemometer was passed through a DISA 55D10 linearizer to a Thermionic Instruments T3000 four-channel tape recorder. The recorded signal was subsequently analysed on an IBM 1130 computer. Input to the computer was through a Krohn Hite fourth-order Butterworth low-pass filter and WDV analog-to-digital converter. The upper cut-off frequency of the filter was usually set at 60 Hz and the lower cut-off was 0·2 Hz.

For the measurements with the DISA 55A25 probe, the shaft projected vertically down towards the bed and the wire was aligned at right angles to the direction of oscillation of the bed. The probe was thus equally sensitive to the velocity component u in the direction of oscillation and the velocity component w normal to the bed but very insensitive to the component v parallel to the bed and normal to the direction of oscillation; it measured the resultant of u and w, denoted in this paper by V. The length of the wire was 1·2 mm and its diameter 5 μm.

The DISA 55A29 probe has a wire of length 1·7 mm and diameter 5 μm inclined at an angle of 45° to the shaft. The shaft projected vertically down towards the bed and the plane containing the shaft and the wire was parallel to the direction of oscillation. Usually, the output was recorded for given values of the stroke and period for about 50 cycles. The probe was then rotated about its shaft through 180°, the motor was stopped to allow the new distance of the probe from the bed to be determined, and the apparatus was restarted with the same stroke and as nearly the same period as possible. A second recording was then made. In no case did the period during the two recordings differ by more than 2%. Provided that the sensitivity of the probe to the component of velocity parallel to the wire is small and that the horizontal component of velocity perpendicular to the direction of oscillation is small compared with the other components, it may readily be shown (see, for example, Keiller 1974) that the difference between the squares of the velocities recorded with the probe facing in opposite directions is $2uw$. The errors involved in making the above assumptions in the present case are discussed by Keiller (1974), who estimates that they probably do not exceed about 6 %.

With the beds of spheres both probes were positioned such that they were directly over the crest of a sphere at mid-stroke. With the gravel the location was arbitrary.

The probes were calibrated by oscillating them in still air at various frequencies and amplitudes. The output from the anemometer was recorded on magnetic tape and then fed into the computer in the way described above. The amplitude of the velocity of oscillation was calculated from the measured stroke and frequency. The resulting calibration curve relating the amplitudes of the anemometer output and probe velocity was then split into short sections each of which was approximated by a straight line or a segment of ellipse in order to facilitate the computer analysis of the test results. It might seem from this description that the use of a linearizer was unnecessary. However it was observed that, without the linearizer, the curve relating velocity to anemometer output showed some dependence on plate acceleration. This effect could be almost totally eliminated, over the range of conditions for which tests were carried out, by suitable adjustment of the linearizer.

It is well known that the curve obtained when a hot-wire anemometer is calibrated at large distances from solid boundaries may not be applicable to measurements made very close to a wall. For the tests described below, Keiller (1974) estimates that the wall effect will be negligible at distances z greater than 0·5 mm from the bed. The corresponding value of βz in these tests is between 0·2 and 0·3.

The speed of the tape recorder and the rate at which the computer sampled the record were varied according to the period of oscillation so as to allow between 111 and 170 velocity measurements to be made during each cycle. The rounding-off error when the sampled signal was converted to digital form was 0·2 % of the full-scale voltage. This makes little difference to the maximum velocity but is not insignificant near the point at which the velocity reverses because of the nonlinearity of the calibration curve. This explains the irregularity of the records shown in figures 2, 10 and 12 at very low velocities. The period of oscillation was determined both by direct measurement using a stop-watch and by the computer from the elapsed time between signals from a phase marker. This marker consisted essentially of a photo-transistor which was switched on and off by the passage of the plate in front of a constant light source. Output from the marker was recorded on magnetic tape at the same time as that from the anemometer. The difference between the two estimates of the period was in no case greater than 2 %. The amplitude U_0 of the bed velocity shown in the various figures was always calculated from the period recorded by the computer.

The measurements presented below represent the average of results for 30 recorded cycles. This was really only necessary when dealing with measurements in turbulent flow but it was found convenient to adopt the same procedure in all cases.

3. Velocity measurements

3.1. *Smooth plate*

A number of tests were made with a bed consisting of a smooth plate of glass in order to provide a check on the experimental arrangement. Figure 1 shows an example of the measured variation in amplitude and phase of the maximum velocity with distance above the bed. The agreement between the measurements and Stokes' theoretical solution for a smooth flat plate is satisfactory.

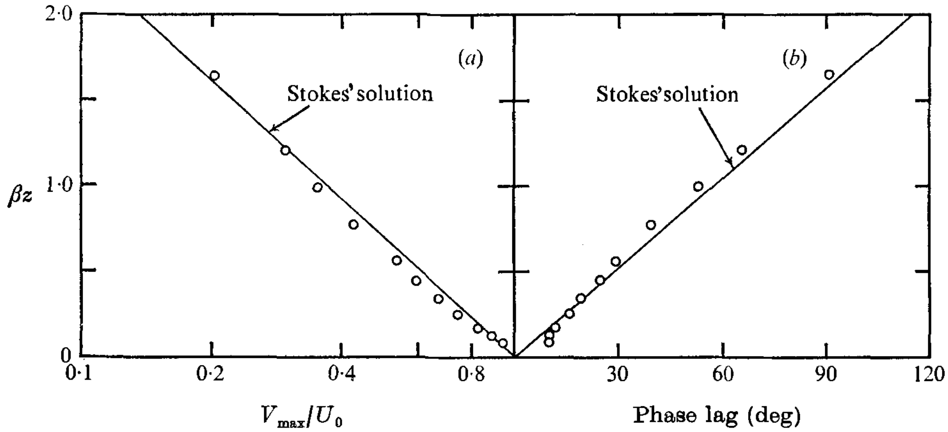

FIGURE 1. The variation in velocity with height above a smooth flat plate. (*a*) Amplitude. (*b*) Phase. $U_0/(\omega\nu)^{\frac{1}{2}} = 49$.

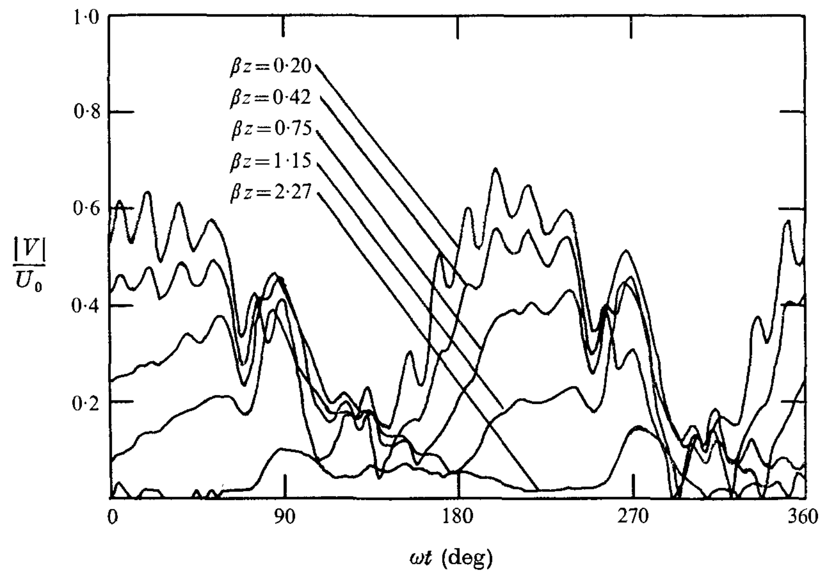

FIGURE 2. The variation in velocity during the course of one cycle at various distances above the bed. $U_0/\omega D = 6{\cdot}87$, $\beta D = 6{\cdot}95$.

3.2. *Beds of spheres*

An example of the way in which the resultant velocity measured by the probe varies during the course of a cycle is shown in figure 2. As a crest of bed roughness passes under the probe the velocity rises and over a trough the velocity falls. The curves shown in figure 2 are the recorded output, uncorrected for the phase shift produced by the filter. This phase shift is not entirely negligible at high frequencies and this is the reason why maxima occur slightly after $\omega t = 0$, when the probe is directly above a crest. Since the phase shift varies with frequency it is necessary, in order to make a correction, to decompose the

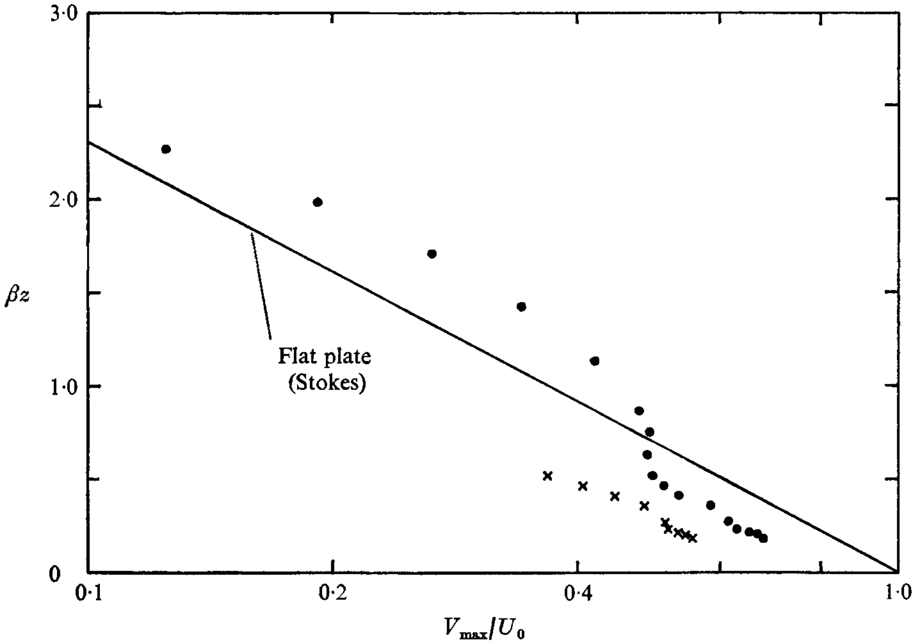

FIGURE 3. The variation with height of the maximum velocity above a crest (circles) and of the velocity above a trough (crosses) at the same instant. $U_0/\omega D = 6{\cdot}87$, $\beta D = 6{\cdot}95$.

signal into its Fourier components, apply the appropriate correction to each component separately and then re-constitute the signal. When this is done it is found that the maxima do occur at $\omega t = 0$ for all values of βz for which the high-frequency oscillation produced by bed roughness can be detected. Here z is measured vertically upwards from the crest of the spheres. The appropriate phase correction has been calculated for the various results presented below and was found to be negligible except in the case of figures 10 and 12. Since these two figures are only included for purposes of discussion it was considered more meaningful to present the recorded output rather than the re-constituted signal.

It should be noted that $1/\beta$ can be thought of as a characteristic distance for viscous diffusion, and on a smooth flat plate in laminar flow could be taken as the boundary-layer thickness; the parameters βD and βz are the sphere diameter and the height z made dimensionless with that length. The other dimensionless parameters to which reference is made below are the Strouhal number $U_0/\omega D$ and the Reynolds number $U_0 D/\nu$. The value of $U_0/\omega D$ is 0·866 times the number of spheres passing the hot wire during one stroke and $U_0 D/\nu$ is equal to $2(\beta D)^2 U_0/\omega D$. These groups may, clearly, be obtained directly by dimensional analysis. For a given horizontal position and fixed bed geometry the velocity V is a function only of z, ν, D, U_0 and ω. Consequently V/U_0 is uniquely determined by βz, βD and $U_0/\omega D$.

Apart from the high-frequency oscillation associated with the passing of individual spheres, the velocity appears to vary more-or-less sinusoidally with

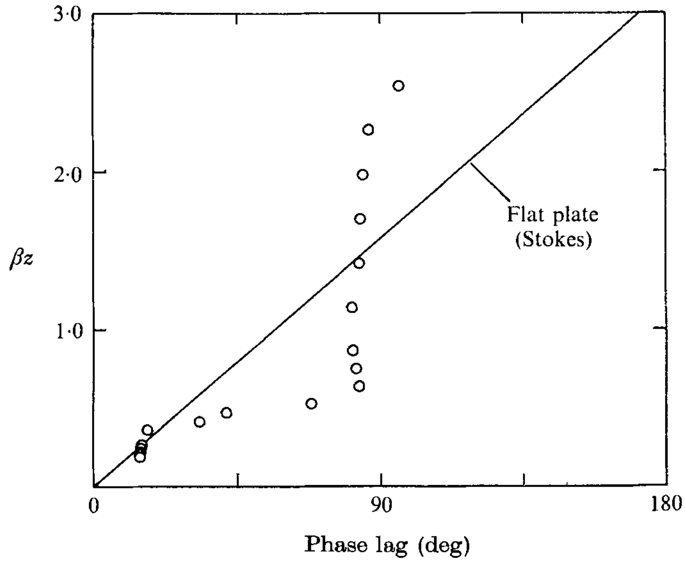

FIGURE 4. The variation in phase of the maximum velocity with height above the bed. $U_0/\omega D = 6{\cdot}87$, $\beta D = 6{\cdot}95$.

time except close to $\omega t = 90°$ and $\omega t = 270°$, where there is a clearly defined secondary peak. At these instants the velocity of the plate is zero and, for the conditions shown in figure 2, the probe is directly over the crest of a sphere. The effect of this secondary peak on the variation with height of the magnitude and phase of the maximum velocity is illustrated by figures 3 and 4. In figure 3 an attempt has been made to obtain separate values for the velocities over a crest and a trough of bed roughness. The maximum recorded velocity is assumed to be the maximum above a crest and the average of the two adjacent minima is assumed to give the velocity above a trough at the same instant. These assumptions are clearly only strictly true in the limit as $U_0/\omega D \to \infty$, but are probably a reasonable approximation in the present case, since the magnitude of the velocity at a fixed point in space varies only slowly with time near its maximum.

Both figure 3 and figure 4 show a marked change in the vicinity of $\beta z = 0{\cdot}6$. Below this point the magnitude and phase of the maximum velocity vary rapidly with height. Above it, the maximum velocity is that at the peak at $\omega t = 90°$ and $\omega t = 270°$. The phase of this peak remains almost constant with increasing height and the variation in amplitude is also different from that closer to the bed.

This secondary peak is clearly of considerable importance and will consequently be examined separately from the velocity during the rest of the cycle.

The secondary peak. Figure 5 shows the limiting values of z/D for which the maximum velocity during the cycle is that at the secondary peak. These limits were estimated by observing the values of z/D between which the phase of the maximum velocity remains approximately constant in the vicinity of $\omega t = 90°$. Because of experimental inaccuracy the transition is not very clearly defined

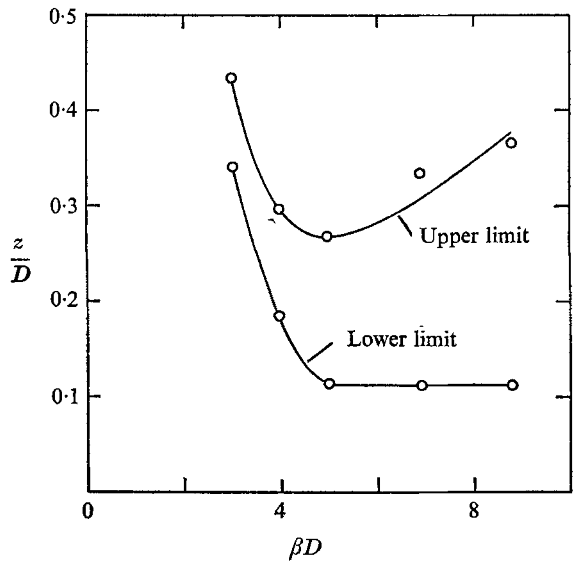

FIGURE 5. Limiting values of z/D for which the velocity at the secondary peak is the maximum.

and consequently only the average for all tests at each value of βD is shown. It is possible, particularly at large βD, that the limiting values of z/D also vary with $U_0/\omega D$. Unfortunately, too few tests were carried out with large values of $U_0/\omega D$ and βD for this to be investigated. It was not possible to detect a secondary peak in the velocity records for $\beta D \leqslant 2{\cdot}0$. No measurements were made in the range $2{\cdot}0 < \beta D < 3{\cdot}0$.

Insight into the nature of this peak may be obtained by examination of the calculations made by Sleath (1974*a*, *b*) for two-dimensional roughness. For values of $\beta D < 7$, for the bed considered by Sleath, the calculations show that sudden jets of fluid occur, up at a trough and down over a crest, at a particular instant in the wave cycle (see, for example, figure 5 of Sleath 1974*a*). The sudden jump in the vertical component of velocity associated with these jets occurs at almost the same instant at different distances above the bed, the phase varying with the value of βD. For $\beta D = 1{\cdot}88$ it occurs at about $\omega t = 60°$, $240°$ and for larger βD progressively later in the cycle. The magnitude of the vertical velocities involved is very small at $\beta D = 1{\cdot}88$ but rises rapidly as βD increases. This would explain why there is no sign of this peak in the measurements for $\beta D < 2{\cdot}0$. With the two-dimensional bed, as βD increases beyond 7 the jets of fluid become progressively less clearly defined, i.e. the velocity rises and falls less sharply with time, and eventually the instability develops into vortex formation. The jets at small values of βD are thus attributable to incipient vortex formation.

Although the flow over a bed of spheres is significantly different from that over a two-dimensionally rough bed, similar phenomena to those described above probably occur. In order to investigate whether that is so, the beds of

Test	β (m^{-1})	βD	$\frac{U_0}{\omega D}$	$\left(\frac{V}{U_0}\right)_{max}$	βz at maximum	Phase at maximum (deg)
31	400	4·94	5·19	0·26	0·62	92
32	403	4·97	6·86	0·35	0·62	89
34	404	4·99	12·22	0·44	0·55	81
39	565	6·96	3·41	0·30	0·87	100
40	561	6·93	5·09	0·41	0·76	95
41	562	6·95	6·87	0·49	0·70	88
42	558	6·88	8·68	0·41	1·16	83
47	472	8·78	3·57	0·41	0·91	100
48	472	8·78	5·35	0·47	1·34	89
51	629	11·69	1·78	0·35	1·78	115
55	403	15·25	1·75	0·52	1·50	116

TABLE 1. Tests in which the secondary peak near $\omega t = 90°$, $270°$ shows a clear maximum as βz varies

spheres were oscillated in still water and crystals of dye dropped onto them. It was observed that tongues of dye were thrown up from the bed at $\omega t = 90°$ and $\omega t = 270°$, i.e. at the instants at which the secondary peaks appear in figure 2. Further confirmation that these secondary peaks are associated with strong vertical velocities is provided by the measurements of the velocity products to be discussed in §4. It seems probable, then, that the secondary peaks observed correspond to strong vertical velocities produced by incipient vortex formation around the spheres on the bed.

As βD increases, the thickness of the Stokes layer corresponds to progressively smaller values of z/D. Since the length scale of vortex formation is proportional to D we should expect the upper curve in figure 5, which represents the value of z/D beyond which the secondary peak ceases to dominate the velocity during the cycle, to continue to rise and the lower limit to fall towards zero as $\beta D \to \infty$.

For given test conditions, the magnitude of the secondary peak rises with increasing βz from zero at the bed to a maximum and then falls off again. The apparent step in the velocity profile above a crest of bed roughness shown in figure 3 is, in fact, a segment in the vicinity of the maximum of the curve representing the magnitude of the secondary peak. Table 1 gives details of those tests in which the maximum was clearly defined. In other tests the maximum was too weak or too near the bed for it to be separated from the nearly sinusoidal component of the velocity. The value of βz at which the maximum occurs is plotted in figure 6 together with some measurements of the product uw, which will be discussed in §4. Although there is considerable scatter the correlation with βD is clearly marked. The straight line is

$$(\beta z)_{\text{at max}} = 0{\cdot}12\beta D. \tag{1}$$

Since all of these tests correspond to βD large compared with unity, it is not surprising that the height of the maximum should apparently be independent of β.

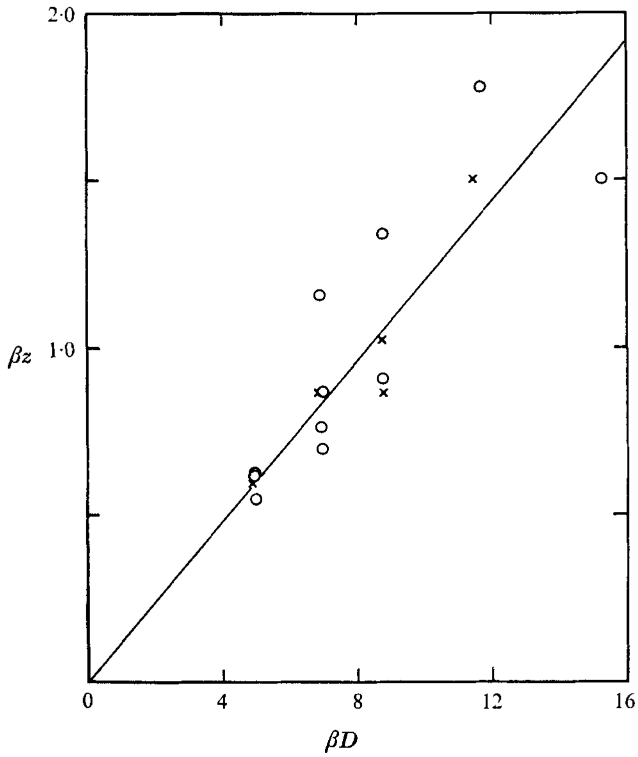

FIGURE 6. The variation with βD of the height at which the maximum velocity at the secondary peak (circles) and the maximum value of uw/U_0^2 (crosses) occur.

From table 1, the magnitude of the maximum tends to increase with βD at constant $U_0/\omega D$, as was found with the two-dimensional bed roughness by Sleath (1974*a*). At constant βD, the magnitude of the maximum increases considerably with increasing $U_0/\omega D$, which is similar to the calculated behaviour for two-dimensional bed roughness, whereas the phase decreases.

The velocity during the remainder of the cycle. The observations with the dye crystals showed that, during the remainder of the cycle, the flow remains closely parallel to the bed.

An example of the variation with height of the amplitude and phase of the velocity is shown in figures 7 and 8. The values in figure 7 of the maximum velocity above a crest and of the velocity at the same instant above a trough have been obtained making the same assumptions as for figure 3. For values of $\beta z > 0{\cdot}2$ the measured points lie parallel to Stokes' theoretical curve for a flat plate, with origin in a crest of bed roughness. In other words, the rough bed may be treated as if it were a flat plate located below the level of the crests of the spheres. Very close to the bed, of course, the effect of the roughness becomes significant as shown by figure 7 (*a*).

The measurements shown in figures 7 and 8 are for values of βD for which it is not possible to detect a secondary peak at all. As illustrated by figure 3, the

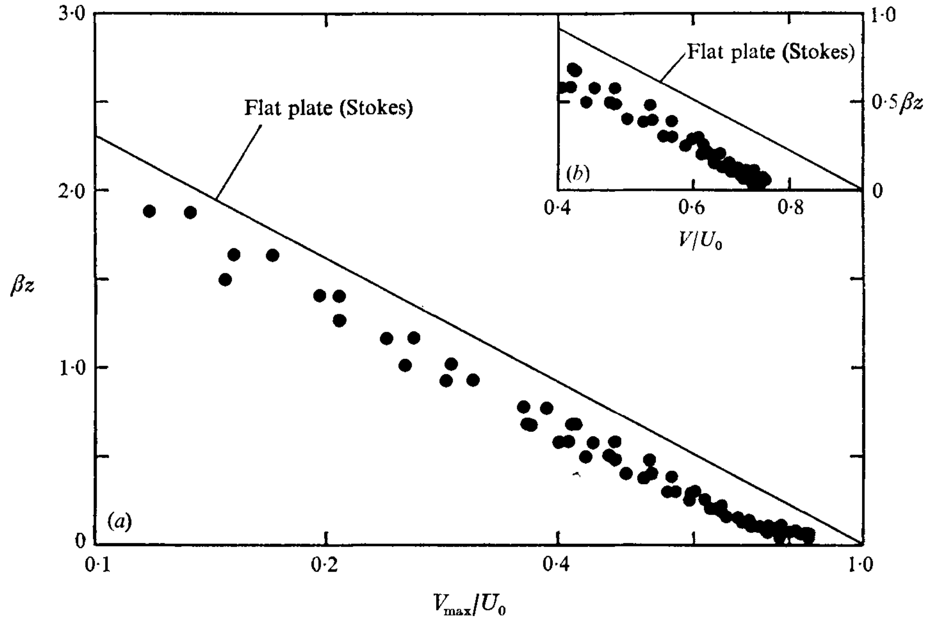

FIGURE 7. The variation in velocity with height for all tests with $1{\cdot}48 \leqslant \beta D \leqslant 1{\cdot}49$. (*a*) Maximum velocity above a crest. (*b*) Velocity above a trough at the same instant.

measurements at larger values of βD show a similar trend in the region close to the bed which is not dominated by the secondary peak. However, for large βD the measurements above a crest and trough do not appear to tend to a common curve in this region. Even at values of $U_0/\omega D$ small enough for the secondary peak to be negligible, it is not to be expected that the velocity distribution at large βD would remain parallel to the flat-plate curve for values of βz greater than about unity. Under these conditions the flow consists of an inner viscous region of thickness of order $1/\beta$ and an outer inviscid region in which the velocity distribution bears no resemblance to the flat-plate solution.

Figure 9 shows the values of the magnitude and phase of the maximum velocity at $\beta z = 0{\cdot}5$, for various values of βD. This value of βz is outside the region dominated by the secondary peak but large enough for the trend of the amplitude measurements to be parallel to the flat-plate curve. The circles represent the average for each value of βD and the side bands show the extreme values for all tests for which $U_0/\omega D > 3$. For $\beta D < 4$ the scatter about the mean appeared to be random but at larger values of βD the dependence on $U_0/\omega D$ was increasingly marked. This is only to be expected since the magnitude of the secondary peak varies with $U_0/\omega D$ and must, in turn, affect the velocity distribution during the remainder of the cycle. Unfortunately, insufficient measurements were made with $U_0/\omega D > 3$ to allow systematic study of the effect of this parameter.

The dashed curves in figure 9 are the theoretical values given by the flat-plate solution when the origin is taken $0{\cdot}12D$ below the level of the crests. The mean values lie reasonably close to these curves for $\beta D \leqslant 5$. The tests at $\beta D = 3$

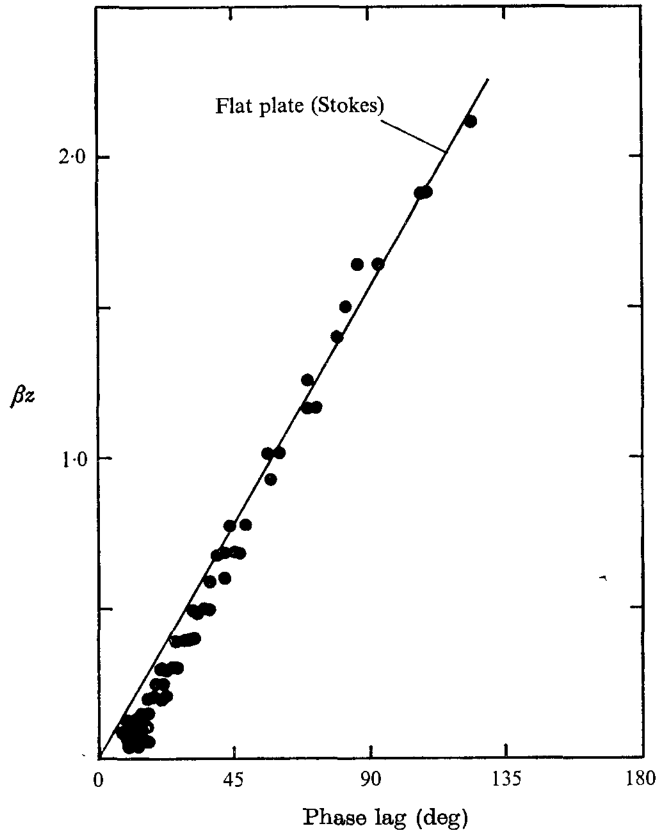

FIGURE 8. The variation in phase of the maximum velocity with height above the bed for all tests with $1{\cdot}48 \leqslant \beta D \leqslant 1{\cdot}49$.

and $\beta D = 4$ were carried out with the spheres of diameter 6·32 mm, which covered only the central portion of the plate. It is possible that these measurements were affected to some extent by end conditions.

At very large values of βD and very low values of $U_0/\omega D$ the viscous layer is very thin and the flow is almost parallel to the surface of the bed. Consequently the velocity distribution close to the bed is accurately given by the flat-plate solution with origin in the local bed surface. This does not, of course, mean that in the absence of Reynolds number effects the value of V_{max}/U_0 at $\beta z = 0{\cdot}5$ will tend to 0·61 above a crest and zero above a trough as $\beta D \rightarrow \infty$, since the velocity outside the viscous layer is not zero.

No attempt will be made here to determine empirical expressions for the velocity distributions in the various regions of the flow. More tests are needed and, in any case, such expressions would be valid only for this particular type of bed roughness and could not be expected to hold for beds of sand and gravel. However, empirical relations are given by Keiller (1974).

Most of the tests were made in purely laminar flow, in the sense that the velocity was observed to be identical from one cycle to the next. This, for

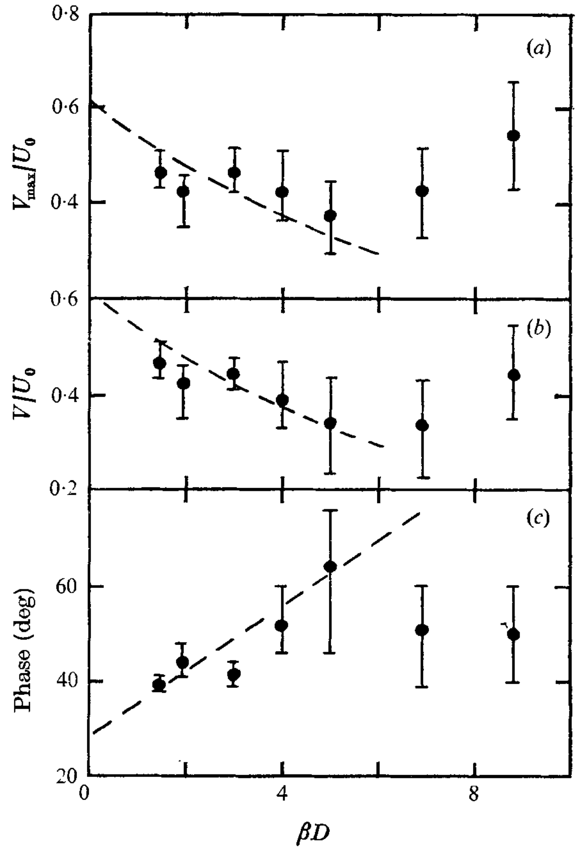

FIGURE 9. Values of the amplitude and phase of the velocity at $\beta z = 0{\cdot}5$ for all tests with $U_0/\omega D > 3$. (a) Amplitude of the maximum velocity above a crest. (b) Amplitude above a trough at the same instant. (c) Phase of the maximum recorded velocity.

example, was the case for the records reproduced in figures 2 and 10. For a few, however, the flow was turbulent. It is not possible to distinguish any sudden change in the mean velocity profile as the flow changes from laminar to turbulent. This suggests that during the initial stages of transition to turbulence the mean velocity profile continues to be determined mainly by laminar processes such as vortex formation and decay. A gradual transition from laminar to turbulent flow was also observed, for two-dimensional bed roughness, by Sleath (1975).

3.3. *Beds of gravel*

An example of the way in which the mean velocity varies during the course of a cycle is illustrated in figure 10. The corresponding measurements of maximum velocity are given in figure 11. In this case it is not possible to separate the velocity above a crest of the roughness from that above a trough. Also, the origin of z is now the highest point on the bed which passes under the probe rather than the level of the crests of the spheres.

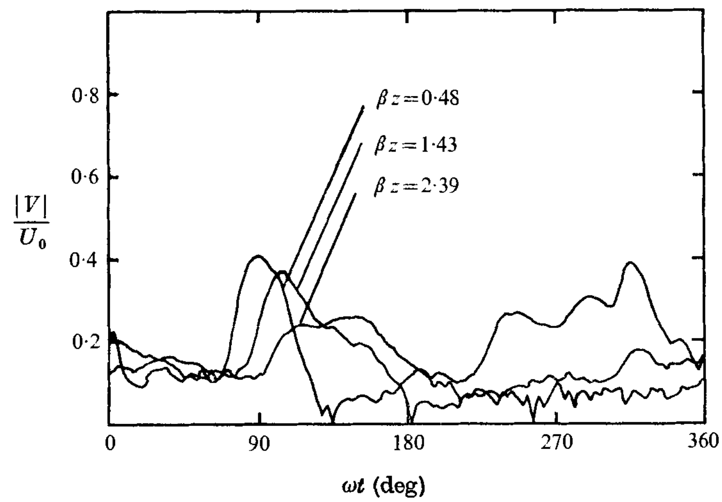

FIGURE 10. The variation in velocity during the course of one cycle at various distances above a bed of gravel. $U_0/\omega D = 6{\cdot}99$, $\beta D = 3{\cdot}42$.

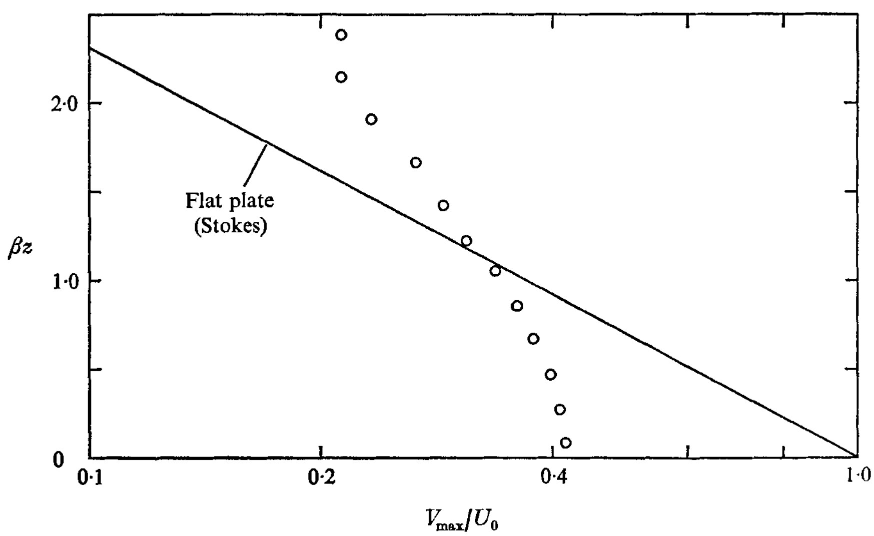

FIGURE 11. The variation with height of the maximum velocity above a bed of gravel. $U_0/\omega D = 6{\cdot}99$, $\beta D = 3{\cdot}42$.

There is some sign in figure 10 of the peak in the velocity near $\omega t = 90°$, but in the vicinity of $\omega t = 270°$ the record is much more irregular. It is not surprising that the record should be different during the two half-cycles since the resistance presented by the bed roughness to the flow is not the same in the two directions of oscillation and the bed does not have the same shape in the region under the probe during each half-cycle. The flow close to the bed must be qualitatively similar to that observed with the spheres but the irregularity of the bed confuses the picture. Also, the probe cannot be taken so close to the bed.

Test	βD	$\frac{U_0 D}{\nu}$	Measured X	X from equation (3)	X from equation (4)
65	6·70	242	4·5	0·9	3·4
66	5·17	244	3·7	0·9	3·0
67	4·53	330	3·1	1·2	3·3
68	3·42	163	2·9	0·6	1·9
69	2·90	211	2·3	0·8	2·0

TABLE 2. Tests with gravel beds

The velocity measurements are very similar to those of Kalkanis (1957, 1964) and Sleath (1970). In particular, when the maximum velocity is plotted in the way shown in figure 11 the measurements lie quite close to a straight line. In other words

$$V_{\max}/U_0 \propto \exp(-\beta z/X), \tag{2}$$

where X is a constant for any given test and is the same as that used by Sleath (1970). The values of X determined for the various tests are listed in table 2. In this table D is the mean diameter of the gravel. It will be noted that the values of $U_0 D/\nu$ lie between those tested by Kalkanis and Sleath whereas the values of βD are here considerably greater.

The relationship obtained by Kalkanis may be written as

$$X = \frac{(U_0 D/\nu)}{266}. \tag{3}$$

The values given by this equation are shown in table 2 together with those calculated from the relationship suggested by Sleath (1970):

$$X = 1 + 0{\cdot}00815(U_0 D^2 \beta/\nu - 115)^{0{\cdot}78}. \tag{4}$$

Of the two relationships, (4) is clearly the more satisfactory but appears to give values of X which are somewhat too low.

The calculations for a two-dimensional bed shape (Sleath 1974*b*) show that when βD is sufficiently large a reasonable approximation for large βz is

$$X = \beta D/2\pi. \tag{5}$$

Since the length scales of a bed of sand or gravel might be expected to be different from those for two-dimensional roughness, the relationship

$$X = \text{constant} \times \beta D \tag{6}$$

might be more appropriate in the present case. This would certainly fit the results given in table 2 quite well. A more detailed comparison between these results, those for the beds of spheres and those obtained by other workers is made by Keiller (1974).

The position of the probe relative to individual roughness elements on the bed was found to have a much greater effect on the phase of the maximum velocity than on its amplitude. Because of this, and the irregularity of the bed,

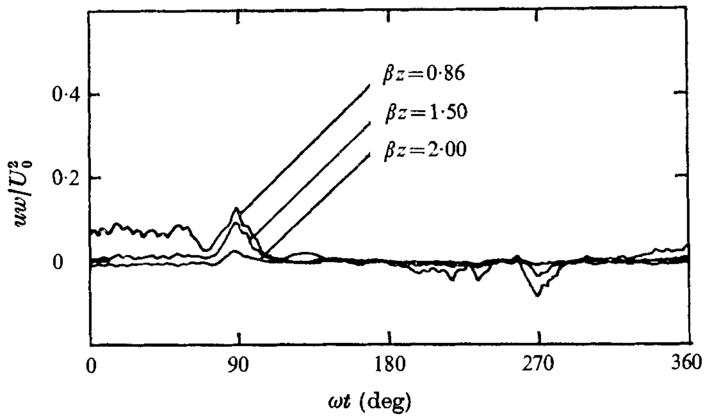

FIGURE 12. The variation in uw during the course of one cycle at various distances above the bed. $U_0/\omega D = 6{\cdot}87$, $\beta D = 6{\cdot}79$.

the variation of phase with height differs from one position of the probe to the next. Consequently phase measurements will not be presented here. The measurements of phase made by Kalkanis and by Sleath did not suffer from this defect because the velocity probes in each case were much more bulky and averaged out fluctuations due to the irregularity of the bed.

The flow was laminar in all of the tests with the beds of gravel except test 67.

4. The product uw

Only the beds of spheres were used for these measurements. An example of the way in which uw varies during the course of a cycle is given in figure 12. Since uw was obtained as a small difference between two large measured quantities the record is understandably irregular. However, the peak found in the velocity record near $\omega t = 90°$, $270°$ is again clearly marked. With this probe it was not possible to make measurements so close to the bed. This may be one reason why the phase at which uw had its maximum value, plotted in figure 13, was not observed to vary rapidly with height very close to the bed as the phase of the maximum velocity did. On the other hand, the relative unimportance of uw/U_0^2 at other points in the cycle may also be taken as confirmation of the observation that the flow is predominantly parallel to the bed except in the vicinity of $\omega t = 90°$, $270°$. Neither in figure 13 nor in figure 14, which shows the way in which the maximum value of uw varies with βz, has any attempt been made to obtain separate curves above crests and troughs. However, the shape of the curves is very similar to that observed with the velocity measurements.

Because of the limited accuracy of these measurements only eight tests were made. The main features are summarized in table 3. The variation of $(uw)_{max}$ with βz was similar to that in figure 14 in all of the tests. The value of βz at which uw had its overall maximum value in any one complete test shows a strong correlation with βD. These values are plotted in figure 6 and it would

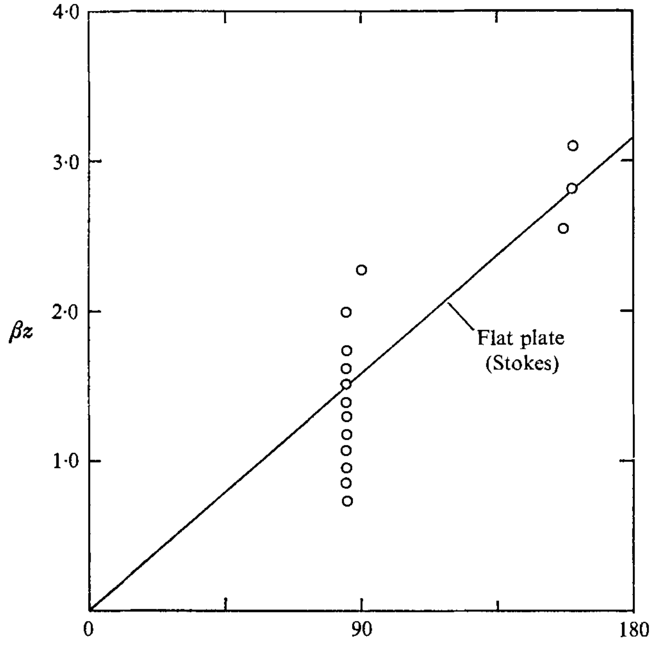

FIGURE 13. The variation in phase of the maximum value of uw/U_0^2 with height above the bed. $U_0/\omega D = 6{\cdot}87$, $\beta D = 6{\cdot}79$.

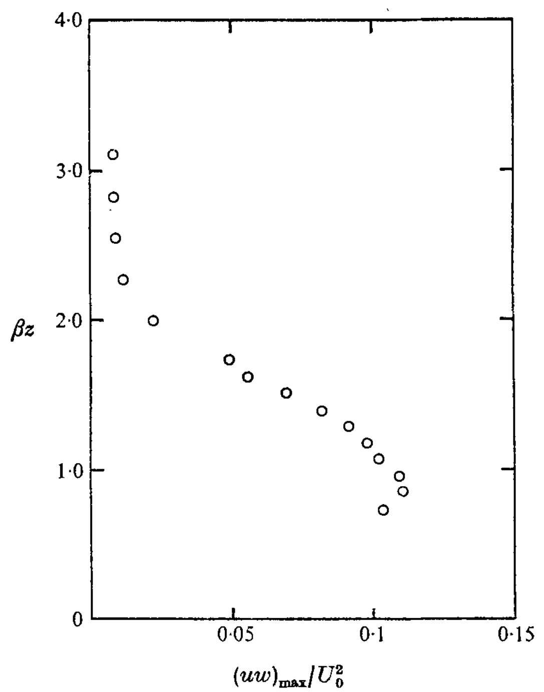

FIGURE 14. The variation with height of the maximum value of uw/U_0^2. $U_0/\omega D = 6{\cdot}87$, $\beta D = 6{\cdot}79$.

Test	β (m^{-1})	βD	$\dfrac{U_0}{\omega D}$	$\left(\dfrac{uw}{U_0^2}\right)_{max}$	βz at maximum	Phase at maximum (deg)
57	399	4·93	3·47	$\geqslant$0·117	—	—
58	397	4·91	6·87	0·061	0·61	−34?
59	400	4·94	12·07	$\geqslant$0·061	—	—
60	550	6·80	3·47	$\geqslant$0·057	—	—
61	549	6·79	6·87	0·111	0·86	89
62	470	8·75	1·78	0·130	0·86	92
63	469	8·74	3·49	0·065	1·02	98
64	615	11·42	1·78	0·094	1·50	107

TABLE 3. The product *uw*

appear that the height at which uw/U_0^2 is maximum is very close to that at which V/U_0 is greatest.

On the other hand, the maximum value of uw/U_0^2 remains reasonably constant as βD varies, although there appears to be some negative correlation with $U_0/\omega D$. The value of uw/U_0^2 quoted is the average of the maxima in each half-cycle. If it is assumed that the third mutually perpendicular component of velocity is negligible at the moment when uw/U_0^2 is maximum, it is possible to estimate the values of u and w by comparison with the measurements of V/U_0. For example, for test 61, the two components of velocity are approximately $0{\cdot}41U_0$ and $0{\cdot}27U_0$. It is not possible to say with certainty which corresponds to u and which to w, but because tongues of dye are observed to be thrown out quite vigorously and because the velocity of the bed is close to zero, it seems likely that the vertical component of velocity at this instant is the larger of the two.

In all of these tests the record was identical from one cycle to the next, i.e. the flow was laminar. It is of interest that purely laminar processes can produce such large velocity products.

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